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LETTER TO THE EDITOR

Central charge for the integrable higher-spin *XXZ* model

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Abstract. The central charge for the integrable higher-spin *XXZ* model is derived, exploiting the exact Bethe ansatz solution by Babujian and Tsvelick. For integer values of ν, where $\gamma = \pi/\nu$ is the anisotropy parameter $(0 < \gamma < \pi/2S)$, we find $c = 3S/(S+1)$, with S the magnitude of the spin. Hence, contrary to expectation, the U(1)-invariant critical point of the theory does *not* renormalise onto a free massless scalar field when $S > \frac{1}{2}$.

The hypothesis that fluctuations at the critical point are not only scale invariant, but also conformally invariant, has led to dramatic progress in the theory of 2D critical phenomena (Cardy 1987b). By exploiting the constraints imposed by conformal invariance (Belavin *et al* 1984), possible classes of critical behaviour can coarsely be indexed by a real number c, the conformal anomaly or *central charge.* In fact, by adding the requirements of reflection positivity of the transfer matrix (unitarity) (Friedan *et al* 1984), and modular invariance of the partition function (Cardy 1986), a complete catalogue of all universality classes with $0 \leqslant c < 1$ can be constructed (Capelli *et al* 1987).

To place restrictions on theories with $c \geqslant 1$, additional symmetries have to be invoked. As pointed out by Affleck (1985), for a theory with a *continuous* internal symmetry group G, the symmetry at the critical point gets enlarged to G$\otimes$G, corresponding to two conserved chiral currents J_+ and J_- of the underlying conformal field theory. In the case of U(1), the most general current algebra consistent with Lorentz invariance (rotation invariance of the statistical system) is given by the Schwinger commutators

$$[J_\pm(x_\pm), J_\pm(x'_\pm)] = \frac{\mathrm{i}}{2\pi}\,\delta'(x_\pm - x'_\pm) \tag{1a}$$

$$[J_+(x_+), J_-(x'_-)] = 0 \tag{1b}$$

where $x_\pm = x_0 \pm x_1$. These follow from representing the chiral currents in terms of a free massless scalar field ϕ,

$$J_+ = -(4\pi)^{-1/2}\partial_+\phi \tag{2a}$$

$$J_- = (4\pi)^{-1/2}\partial_-\phi. \tag{2b}$$

Equations (1*a*) and (1*b*) imply that spacetime translations of J_+ and J_- are generated by the stress tensor with components

$$T_\pm(x_\pm) = \pi J_\pm(x_\pm)J_\pm(x_\pm) + \text{constant}. \tag{3}$$

It follows that the full stress tensor can differ from $T_\pm$ only by terms which commute with the currents. If no hidden symmetries are present, i.e. all charges of the theory are built from J_+ and J_-, dimensional analysis forces the additional terms to have the form J_+J_+ and J_-J_- respectively, leading to a trivial renormalisation of the stress tensor. Hence, barring hidden symmetries, any U(1) critical point can be described by a free massless scalar, with central charge $c=1$.

It is the purpose of this letter to point out the existence of a class of U(1) invariant models with $c>1$. These are the higher-spin generalisations of the well known *XXZ* model and were first constructed and studied by Sogo (1984). The integrability of the theories generates an infinite-dimensional Yang–Baxter algebra (Faddeev 1984), coding the hidden symmetry that apparently drives the central charge away from its expected value. This behaviour suggests a role for the Yang–Baxter algebra in the classification of critical phenomena.

The models can be defined by the Hamiltonian

$$H=J\sum_{n=1}^{N} S_n^xS_{n+1}^x+S_n^yS_{n+1}^y+\cos(2S\gamma)S_n^zS_{n+1}^z+P_{2S}(S_n^xS_{n+1}^x,S_n^yS_{n+1}^y,S_n^zS_{n+1}^z) \qquad (4)$$

where P_{2S} is a polynomial of degree $2S$ containing the higher-order terms (S being the magnitude of the spin) constructed from the vertex model underlying the spin problem (Sogo *et al* 1983, Babujian and Tsvelick 1986). S_n^x, S_n^y and S_n^z are the components of the spin operator at site n, while J is some overall energy scale ($J>0$), and γ is an anisotropy parameter confined to $0<\gamma<\pi/2S$.

As shown rigorously by Babujian and Tsvelick (1986), the Hamiltonian is Bethe ansatz solvable when $\gamma=\pi/\nu$, with ν an integer $>2S$. In particular, in the low-temperature limit the free energy per site can be obtained (Babujian and Tsvelick 1986) and to $O(T^2)$ one finds

$$f=\text{constant}-\frac{1}{J}\frac{S}{S+1}T^2. \qquad (5)$$

By matching (5) to the general expression for the free energy (per volume) of a (1+1)-dimensional conformal theory (Affleck 1986a, Blöte *et al* 1986)

$$f=\text{constant}-\tfrac{1}{6}\pi cT^2 \qquad (6)$$

the central charge can thus be extracted. However, (6) applies to a relativistic theory of massless particles in units where the velocity of light is equal to unity. The 'effective velocity' for the collective excitations in the spin model must therefore be renormalised to unity before a proper matching can be done. As discussed by von Gehlen *et al* (1986), this procedure also guarantees that the equations of motion are conformally invariant.

To identify the effective velocity, the exact dispersion relation for the excitations has to be determined. Constructing the low-lying states from the Bethe ansatz equations in Babujian and Tsvelick (1986), we find a spectrum of solitons and antisolitons carrying spin $\frac{1}{2}$, and with energy-momentum relation (Johannesson 1988):

$$E(k)=\tfrac{1}{2}\pi J\sin k \qquad 0\leqslant k<\pi. \qquad (7)$$

Up to a multiplicative factor, this has the same form as the dispersion relation derived by Sogo (1984) from a differently constructed vertex model defining the generalised *XXZ* problem. Reading off from (7), the effective velocity in the low-energy limit is seen to be equal to $\frac{1}{2}\pi J$. As follows from dimensional analysis, renormalising this velocity to unity in (5) means multiplying by $\frac{1}{2}\pi J$. The temperature term then becomes $\frac{1}{2}\pi[S/(S+1)]T^2$ and comparison with (7) yields

$$c = \frac{3S}{S+1}. \tag{8}$$

This is the same set of c numbers as predicted by Affleck (1986b) for the isotropic XXX model from a mapping onto the SU(2) Wess-Zumino-Witten model (Knizhnik and Zamolodchikov 1984), and verified in Johannesson (1988) using exact finite-size scaling. Curiously, the explicit breaking of SU(2) down to U(1), induced by choosing $\gamma \neq 0$ in (4), leaves the central charge untouched, *provided* γ *is selected from the discrete set* $\gamma = \pi/\nu$, $\nu \in Z$.

Kirillov and Reshetikin (1987a, b) have analysed the model in (4) for *arbitrary* values of the anisotropy parameter in the interval $0 < \gamma < \pi/2S$. Now the free energy will in general get contributions from collective excitations with different effective velocities, i.e. the spectrum decomposes into a collection of distinct Fermi liquids. This corresponds to an unphysical field theory with several distinct 'velocities of light' for which conformal invariance makes no prediction. However, for certain privileged choices of γ only a single Fermi liquid is present. Unfortunately, due to the implicit form of the solution, we have not been able to disentangle the physically relevant quantities except for the case above, already launched by Babujian and Tsvelick (1986).

Of obvious interest is the calculation of the anomalous dimensions of the theory. As known since the pioneering work by Cardy (1984), these can be accessed through a study of the finite-size corrections to the excitation spectrum. For the present model, this will require some rather cumbersome analysis, possibly along the lines of de Vega and Woynarovich (1985). Taking the solution of the well known spin-$\frac{1}{2}$ XXZ model as a guide, where the anomalous dimension of the S^z operator is given by $\eta^z = \frac{1}{2} \times (1-\gamma/\pi)^{-1}$ (Luther and Peschel 1975), it is tempting to conjecture that the higher-spin generalisations will also have critical exponents varying with the anisotropy. If so, the low-energy limit could provide an example of a conformal theory with a 'conspiring' operator product expansion. As recently shown by Cardy (1987a), a fixed line responsible for continuously varying exponents can arise in a theory with $c \neq 1$ only if the coefficients in the operator product expansion 'conspire' to produce precisely the right scaling form for the marginal operator. The necessary fine tuning of the OPE coefficients could naturally arise if hidden symmetries are present, again hinting at a possible role for the Yang-Baxter algebra in influencing the critical behaviour. One should note, however, that it is not clear in what sense the exponents would here be *continuously* varying, considering that a single Fermi liquid is present only for a discrete set of anisotropies.

To conclude, it is certainly desirable to advance the understanding of this intriguing model.

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